

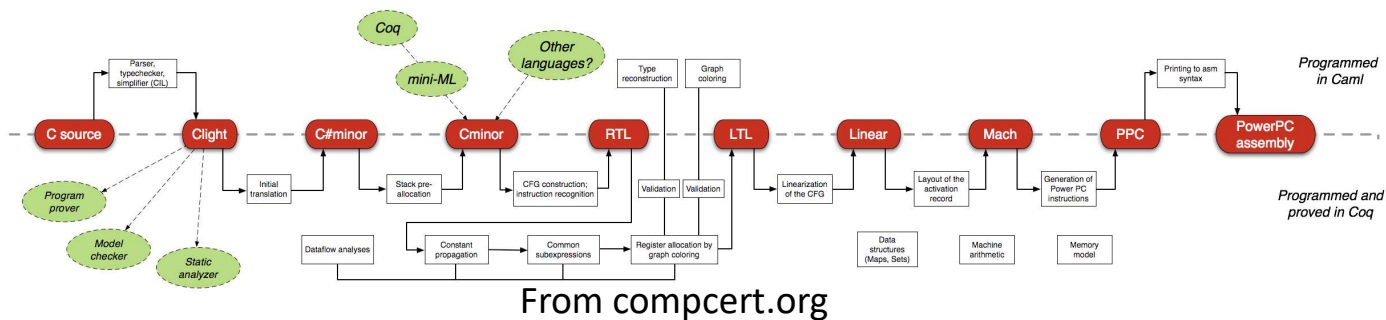
# Fully Verified Instruction Scheduling

**Ziteng Yang, Jun Shirako, and Vivek Sarkar**



# Compiler Correctness & Verification

- **Formal Verification:** seeking 100% correctness guarantee
- **CompCert**'s approach: directly prove correctness in an interactive theorem prover (Coq)
  - Write the compiler as a Coq function<sup>1</sup>
  - Formalize semantics of C, IR and assembly language in Coq
  - Prove semantics preservation of each translation in Coq



**Other research projects: VeLLVM, CakeML, CertiCoq**

<sup>1</sup>In the actual engineering, the Coq function was finally extracted into OCaml function to generate an executable file

# Current state of CompCert

- **Only verified several basic optimizations (O1-level optimization)**
  - e.g. Constant propagation, common subexpression elimination, redundancy elimination
  
- **Only support in-order translation**
  - It cannot reorder instructions a.k.a *instruction scheduling* (O2- and O3-level)

# Motivation for Compiler-level Instruction Scheduling

- Improve instruction-level parallelism and reduce pipeline stall for *in-order processors*. [e.g. Cortex-A53, U74MC]

original order



$i_1$  reads  $r_1$   
 $i_2$  writes  $r_2, r_3$   
 $i_3$  writes  $r_1, r_2$   
 $i_4$  writes  $mem$   
 $i_5$  reads  $r_3$ , writes  $mem$

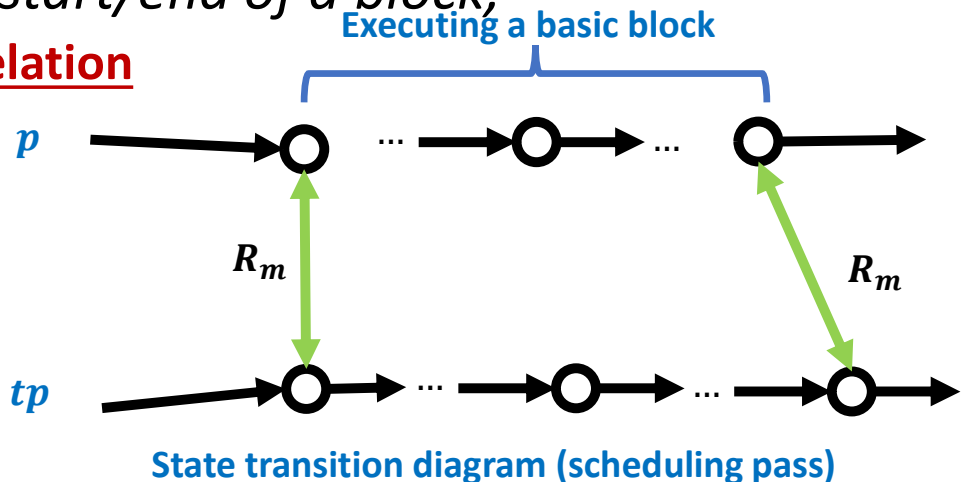
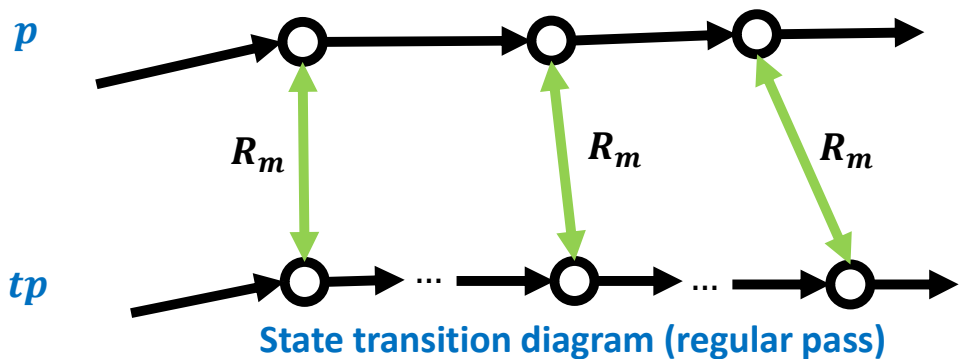
Cycle	ADD	SUB	FLOAT
1	$i_1$		$i_1$
2			$i_1$
3	$i_2$		$i_2$
4			$i_2$
5	$i_3$		$i_3$
6	$i_4$	$i_4$	
7	$i_5$	$i_5$	$i_5$

Cycle	ADD	SUB	FLOAT
1	$i_1$		$i_1$
2	$i_4$	$i_4$	$i_1$
3	$i_2$		$i_2$
4			$i_2$
5	$i_3$		$i_3$
6	$i_5$	$i_5$	$i_5$
7			

# Challenges in Verified Instruction Scheduling (intra-block)

- Semantics details of a reordering instructions
- Program states only *matches at the start/end of a block,*

- no longer a lock-step simulation relation



- Handling dependence relations between each instructions
- Potentially heavy proof workload
- **Previous work:** verified translation validation

10/24/24 • [POPL'08] Tristan, Jean-Baptiste, and Xavier Leroy. "Formal verification of translation validators: a case study on instruction scheduling optimizations."  
 [OOPSLA'20] Six, Cyril, Sylvain Boulmé, and David Monniaux. "Certified and efficient instruction scheduling: application to interlocked VLIW processors."

# Full Verification v.s. Verified Translation Validation

	Full Verification	Verified Translation validation
Algorithm Correctness	Yes	No
Development Difficulty	Harder (Potentially)	Easier
Compile Time Overhead	Lower (develop-time proof)	Higher (compile-time validation)
Flexibility	Lower (May need to change proof when changing algorithm)	Higher (Only validate output with input)
Methods	Proof Assistant + Principle of Algorithm Correctness	Proof Assistant + Symbolic Execution

# Full verification v.s. Verified Translation Validation

Previous work on instruction scheduling [Tristan et al. 2008]  
[Six et al. 2020]



**Final theorem of a verified translation validation:**

$\forall p\ tp, \text{ if } \text{compile\_pass}(p) = tp \wedge \text{validate}(p, tp) = \text{True},$   
*then semantics\\_preserve(p, tp)*

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**Final theorem of a fully verified compilation:**

(harder to prove, but stronger result)

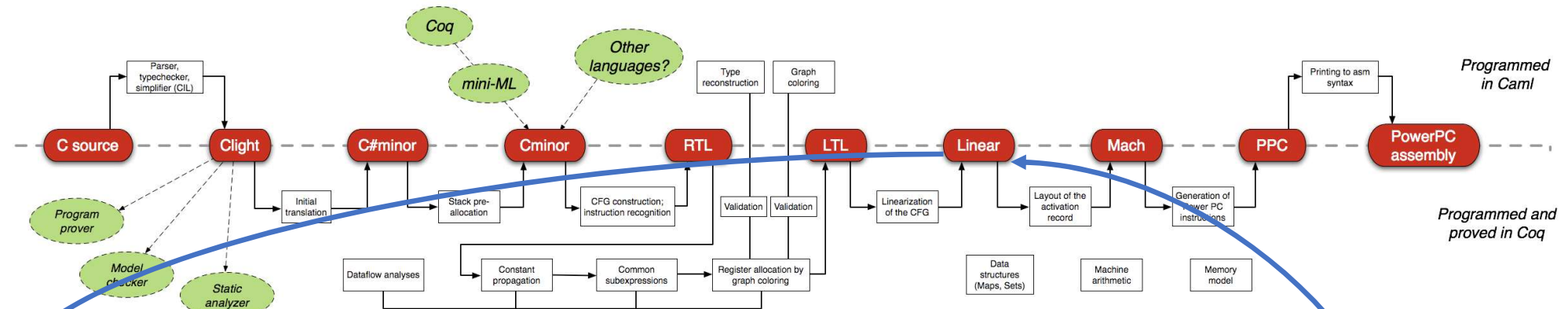
$\forall p\ tp, \text{ if } \text{compile\_pass}(p) = tp,$   
*then semantics\\_preserve(p, tp)*



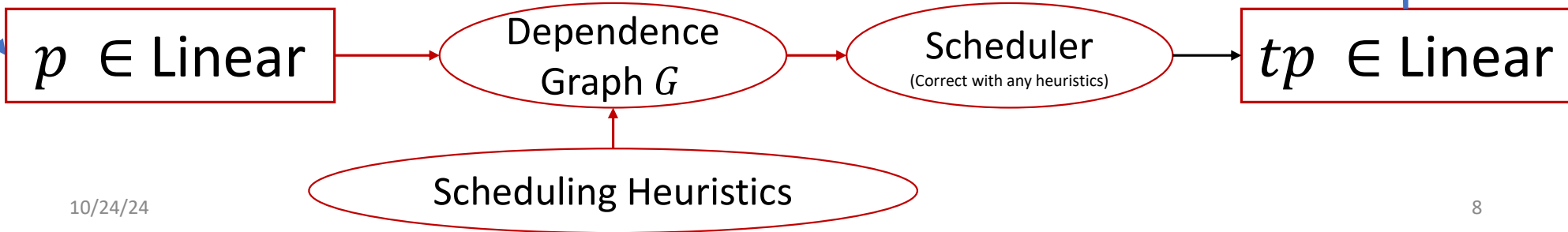
**This project: make this part verified,  
while keep the proof work lightweight**

***a.k.a. correct-by-construction***

# This project



## Instruction Scheduling





# **Proof Logical Chain**

**Part I: swapping lemma : a property of topological order**

**Part II: syntax-level valid instruction scheduler**

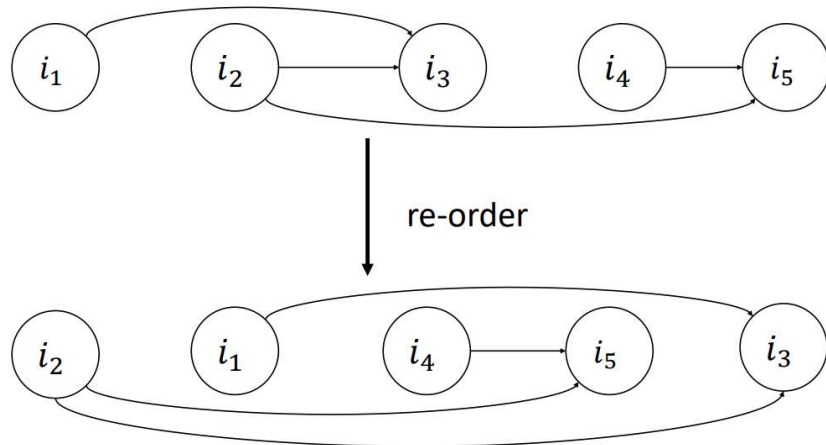
**Part III: decomposing a valid scheduler**

**Part IV: transitivity of semantics preservation**

**Part V: correctness of swapping (semantics level)**

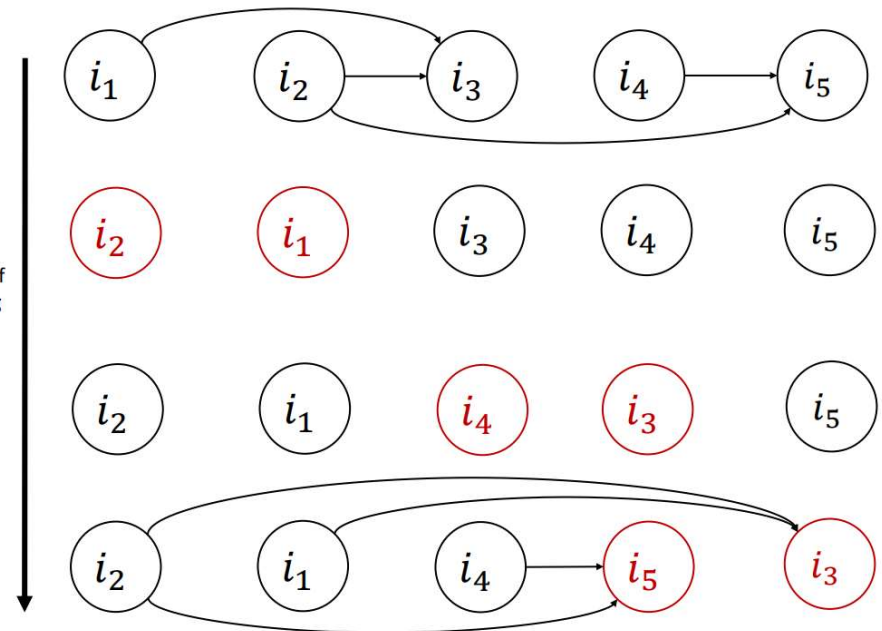
# Part I: swapping lemma- a property of topological order

**Swapping lemma:** A topological reordering of a list of partially ordered elements is equivalent to a finite sequence of **swaps** of **adjacent but not ordered** elements.



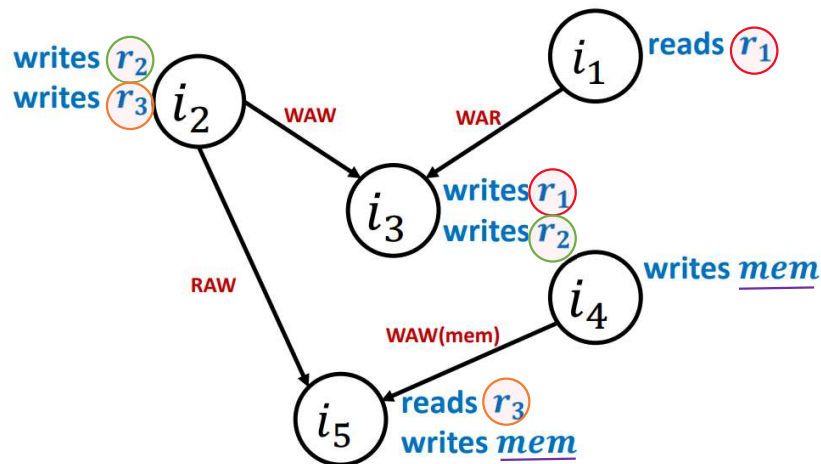
=

a series of  
swapping  
attempt



## Part II: **syntax-level valid** instruction scheduler (intra-block)

The **dependence constraints of the original program**: a valid instruction scheduler conduct a topological reordering based on the **dependence relation** (defined in syntax level, by matching the register name).



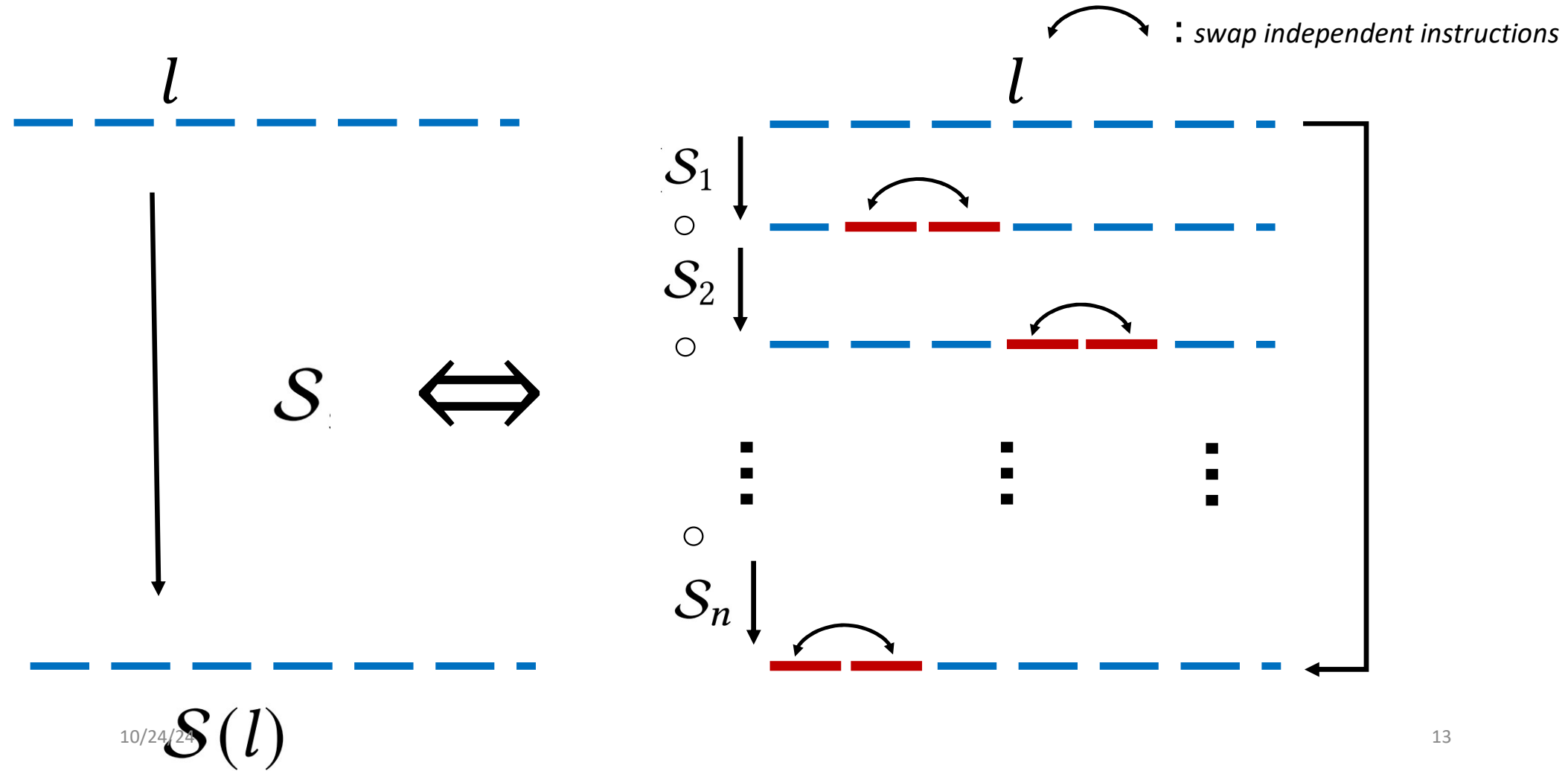
## Part III: decompose a valid scheduler

Any syntax-level valid scheduler,  
reorders a program's instructions

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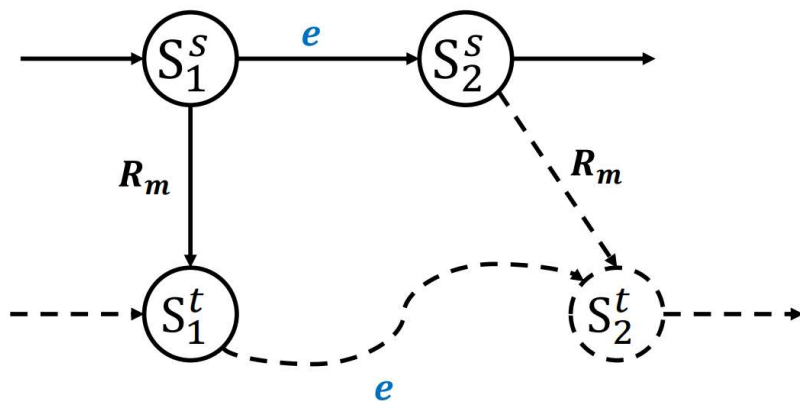
Composition of a **finite sequence** of compiler passes,  
that **only swap one pair** of independent instructions  
(named *single swappers*)

# Decomposing a scheduler



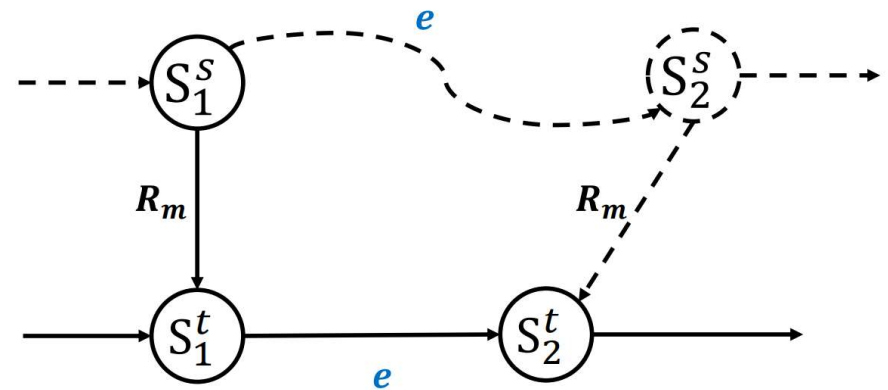
# Part IV: transitivity of semantics preservation

- **The final goal of CompCert proof:** *backward simulation* of state transition between C and compiled Asm program's small-step semantics, through **only proving forward simulation of each pass** and lemmas that “flips” the simulation direction<sup>1</sup>



**Forward simulation** (sufficient to prove this only for each single compiler pass)

implies



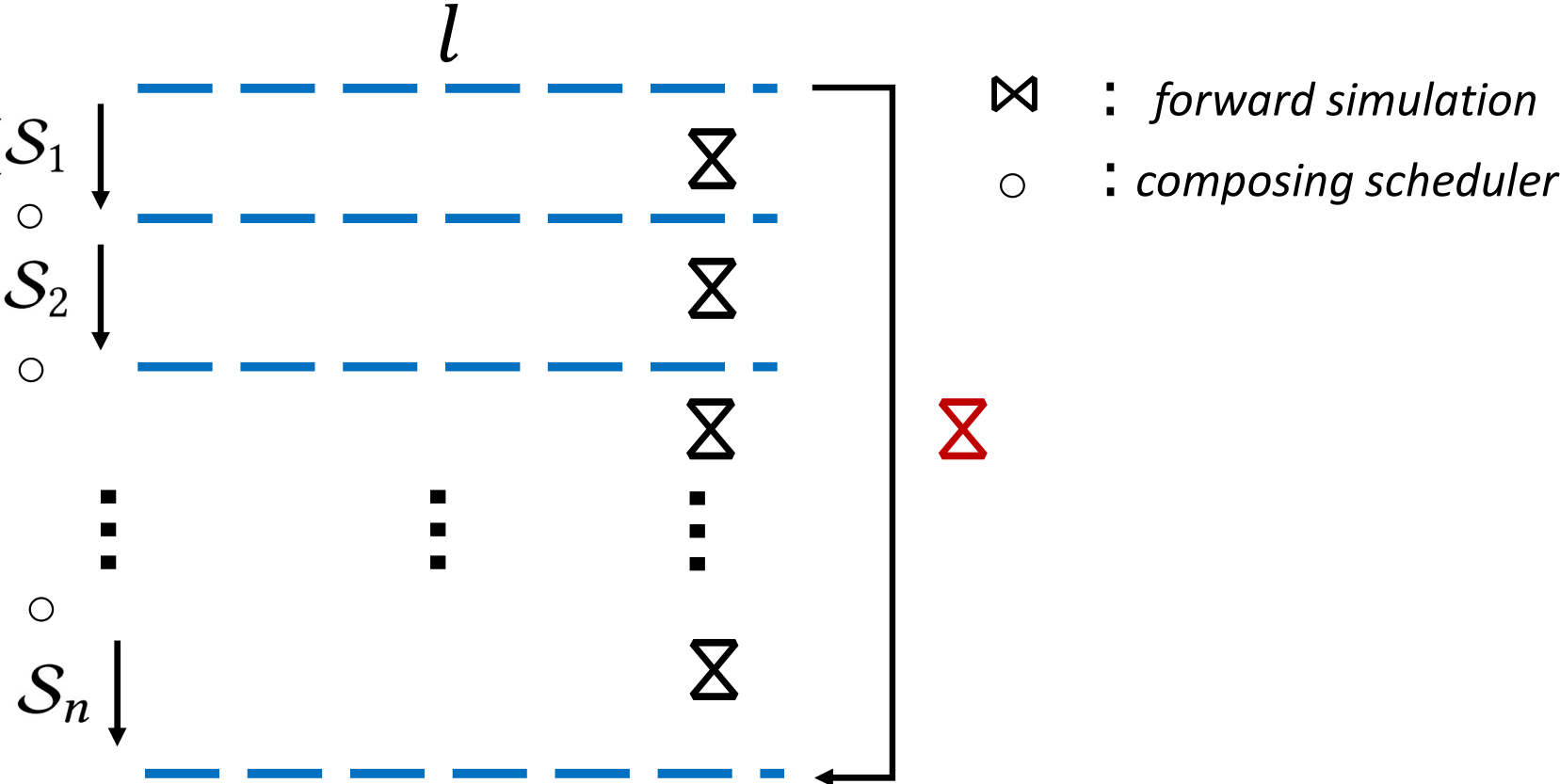
**Backward simulation** (final goal of whole compiler, derived by forward simulation and determinism of assembly language)

<sup>1</sup>10/24/24

<sup>1</sup>One of the base theory of CompCert

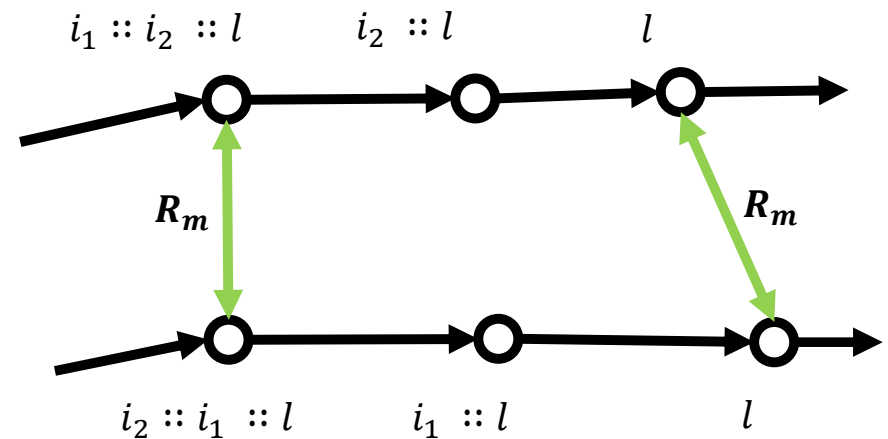
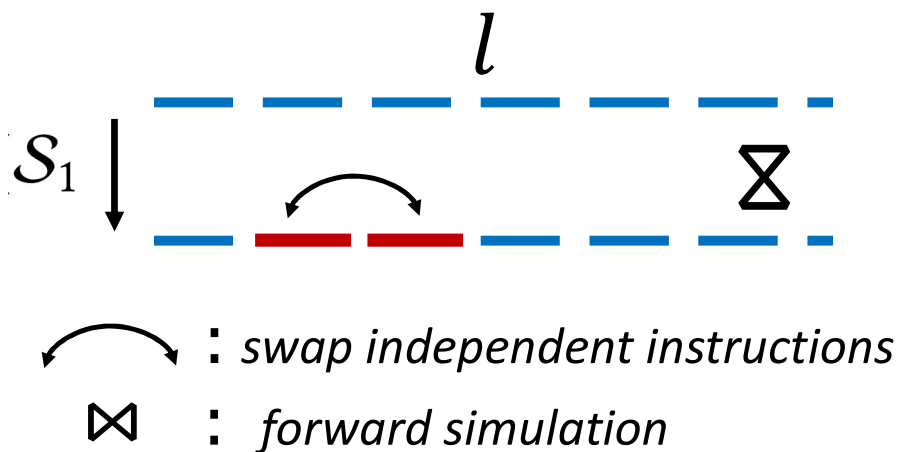
**Lemma:** *forward simulation is transitive*

- one of the base theory of CompCert



# Part V: correctness of swapping

The only lemma that requires reasoning on semantics details: swapping only one pair of adjacent **syntax-level** independent instructions (RAW/WAR/WAW dependence derived by pattern-match) inside only one basic block of a program satisfied the forward simulation, a.k.a. **semantics-level equivalence** of the program





# Whole proof idea

$l$ , with a generated dependence relation  $R$

$S$   $\longleftrightarrow$

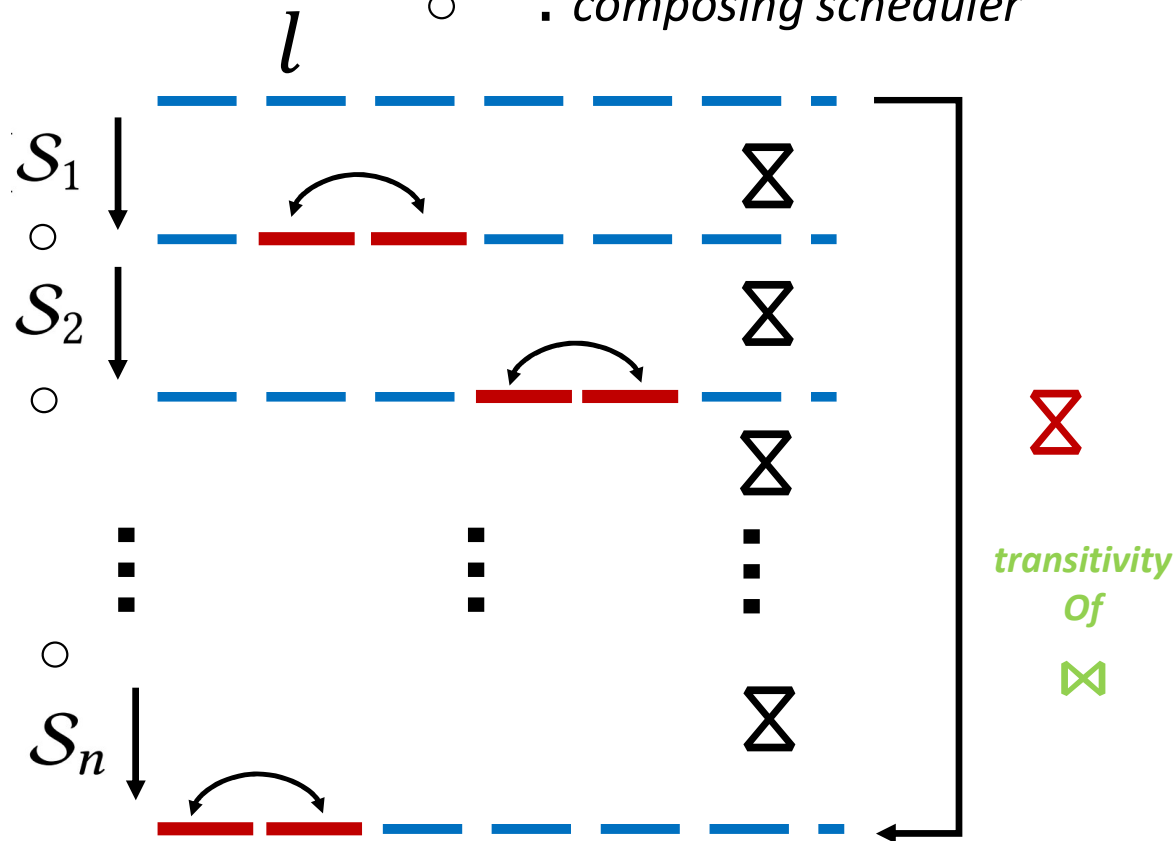
topo-reorder of  $l$  by  $R$

$S(l)$

$\curvearrowright$  : swap independent instructions

$\bowtie$  : forward simulation

$\circ$  : composing scheduler



transitivity  
of  
 $\bowtie$

# What did we get till here?

**A general framework** to prove any instruction scheduling algorithm

- In other words, the theory above is **once-for-all**

The framework was formalized in Coq (based on CompCert framework)

# Prove a list-scheduling using our framework

- **A concrete instruction scheduling implementation**
  - Generate the dependence graph of original basic block
  - Iteratively choose and pop an available instruction, according to an outside **scheduling heuristics** to the scheduled list

---

**Algorithm 1** Dependence Graph Generating:  $DRel(l)$

---

**Require:** List of instructions  $l = [i_1, i_2, \dots, i_n]$  ▷ Non-duplicate by giving index to them

**Ensure:** Graph  $G$  that records  $\mathcal{G}_l^{\mathcal{D}}$ , the generated order of  $l$  by  $\mathcal{D}$  ▷ Proved in Section.5.3

**if**  $l = nil$  **then**

$G.nodes \leftarrow E_l$

$G.edges \leftarrow \emptyset$

**else if**  $l = i' :: l'$  **then**

$G.edges \leftarrow G.edges \cup \{(i', i) \mid i \in l' \wedge \mathcal{D}i'i\}$

$G.edges \leftarrow G.edges \cup DRel(l').edges$

**end if**

---

**Algorithm 2** List Scheduling  $\mathcal{S}^*(\mathcal{P}, l)$

---

**Require:** A heuristic function  $\mathcal{P} : list\ instruction \rightarrow list\ N$

**Require:** List instructions  $l = [i_1, i_2, \dots, i_n]$  ▷ Non-duplicate by giving index to them

**Ensure:**  $l^*$  is a topo-reorder of  $l$  by  $\mathcal{G}_l^{\mathcal{D}}$  ▷ Proved in Section.5.3

$G \leftarrow DRel(l)$

$Priority \leftarrow \mathcal{P}(l)$

▷  $\mathcal{P}(l)(k)$  will the priority of  $i_k$

$l^* \leftarrow []$

**while**  $G$  not empty **do**

$A \leftarrow \{i_k \in l \mid \forall i_{k'} \in l.(i_{k'}, i_k) \notin G\}$

$i_{k^*} \leftarrow i_{k^*} \in A$  such that  $Priority[k^*]$  is max

$l^* \leftarrow l^* ++ [i_{k^*}]$

$G \leftarrow$  remove node  $i_{k^*}$  from  $G$

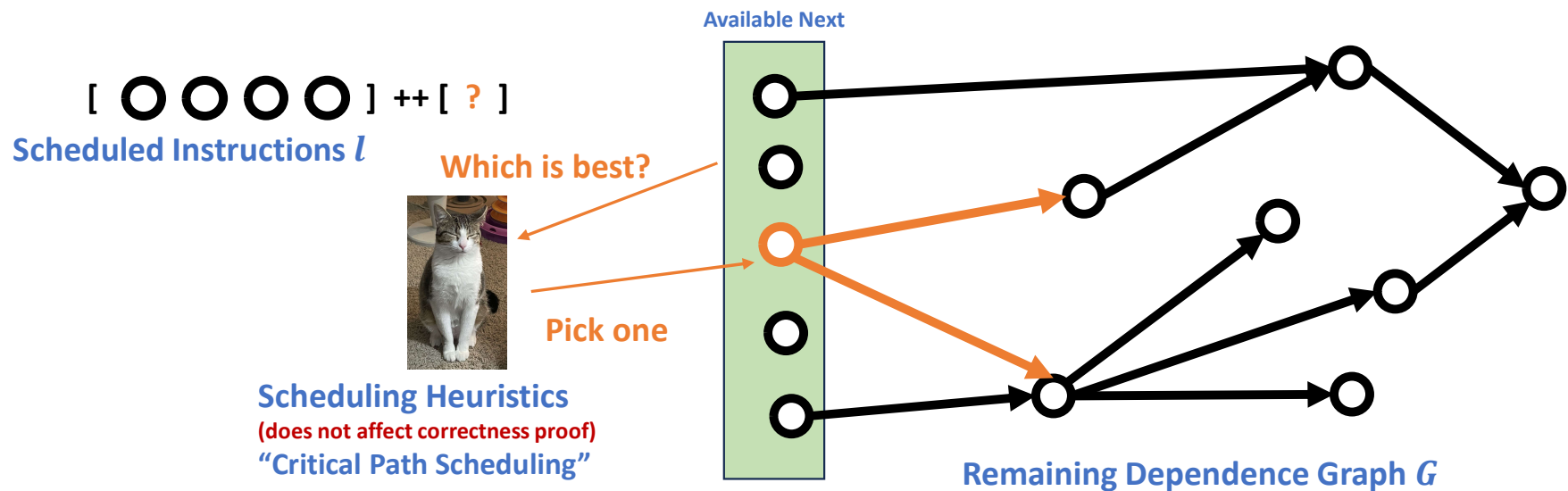
**end while**

**return**  $l^*$

---

# Prove a list-scheduling using our framework

- A concrete instruction scheduling implementation
  - Generate the dependence graph of original basic block
  - Iteratively choose and pop an available instruction, according to an outside *scheduling heuristics* to the scheduled list



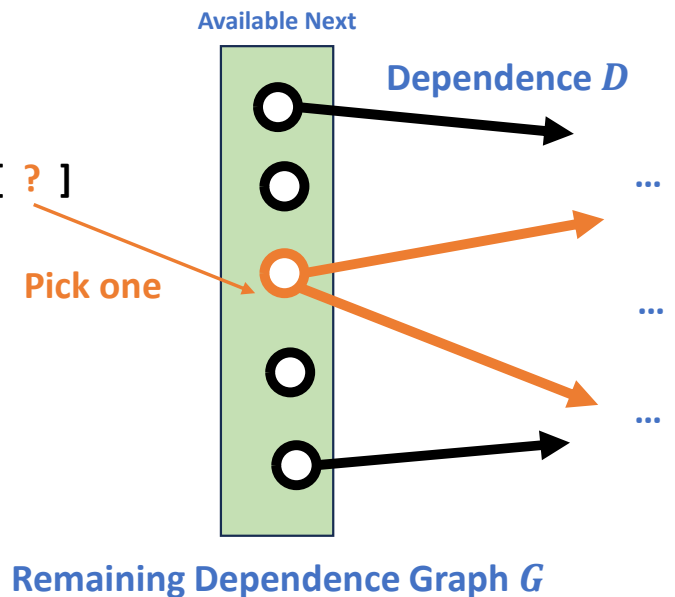
# Prove a list-scheduling using our framework

Brief idea of prove the top-logical reorder:

the scheduler maintains an **invariant** during scheduling

- **EXCLUSIVE:**  $l \cap G.\text{node} = \emptyset$
- **SUB:**  $l \cup G.\text{node} \subseteq \text{original}$
- **SORT:**  $l$  is sorted by  $D$

[ ○ ○ ○ ○ ] ++ [ ? ]  
Scheduled Instructions  $l$



Everything is formalized/checked in Coq,  
and incorporated into CompCert project

# An Evaluation on Proof Engineering

	Language	Functions	Proofs
Base theories on topo-reorder's properties (once-for-all)	Coq	-	0.8k
Base theories on semantics (once-for-all)	Coq	-	2.2k
List-scheduling algorithm (excluding heuristics)	Coq	0.15k	1.0k
Scheduling heuristics	Ocaml	25	-
Scheduling heuristics	C	0.7k	-
Machine dependent proof (Risc-V)	Coq	-	40
Machine dependent proof (x86)	Coq	-	35

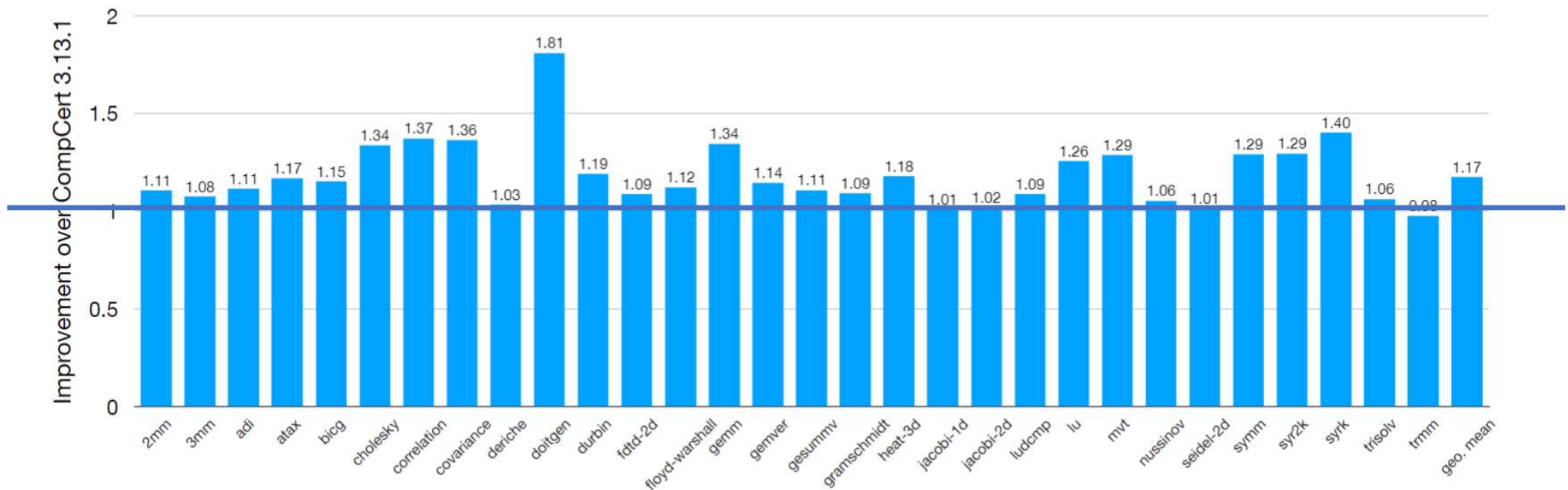
## LOC of program/functions and proofs in our work

	Fully Verified	Scope	Lines of Proof Code
This work	Yes	List Scheduling	4k
[Tristan et al. 2008]	No	List and Trace Scheduling	11k
[Six et al. 2020]	NO	List Scheduling (VLIW)	18k + 10k (architecture)

## LOC of related work

# An Evaluation on Our List-scheduler's Performance

(Improvement in execution times on Risc-V hardware platform)



Performance improvements by the certified instruction scheduler for PolyBench C 4.2

# Future work

- **Verifying Inter-block Scheduler, with alias analysis of memory access**
    - Best existing method also used verified translation validation only
    - Block size change. Swapping lemma won't work
  - **General: towards multi-level parallelism for verified compiler**
    - Data-level parallelism
    - Instruction-level parallelism (this work improved)
    - Task-level parallelism (e.g. loop parallelism)
- a.k.a. bringing CompCert to O2/O3-level optimization



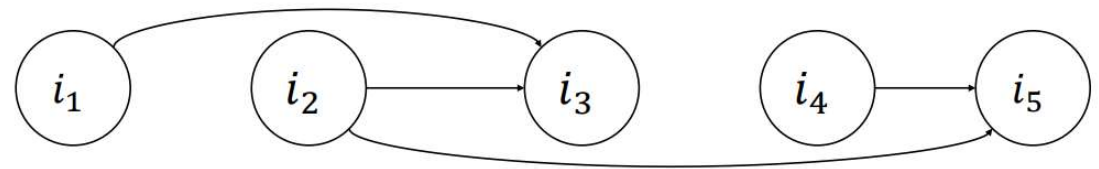
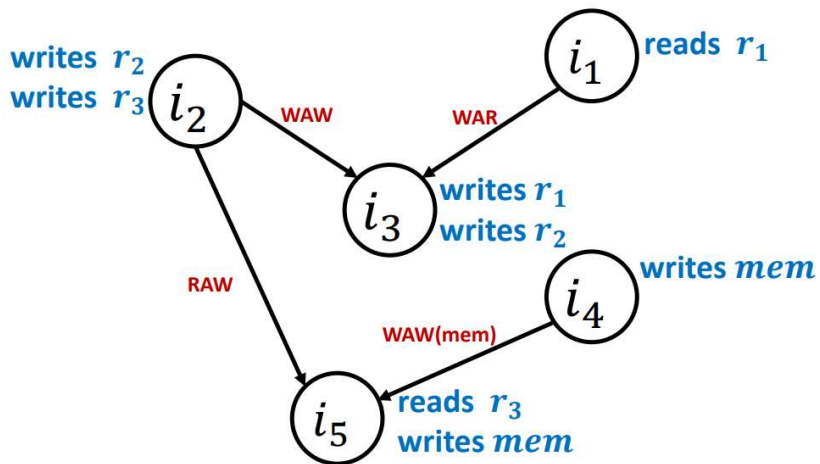
# Summary: verified instruction scheduling framework with multi-level flexibility

- **Flexible algorithm changes**
  - Change algorithm => only change proofs on syntax dependence preservation
  - Nothing about semantics again
- **Flexible instruction scheduling heuristics**
  - Change scheduling heuristics => no change of correctness proof
- **Flexible Machine Architecture**
  - Machine independent implementation (40 lines of Coq code diffs between x86-64/Risc-V)
  - Only implement different heuristics for different architecture
- **Q & A?**

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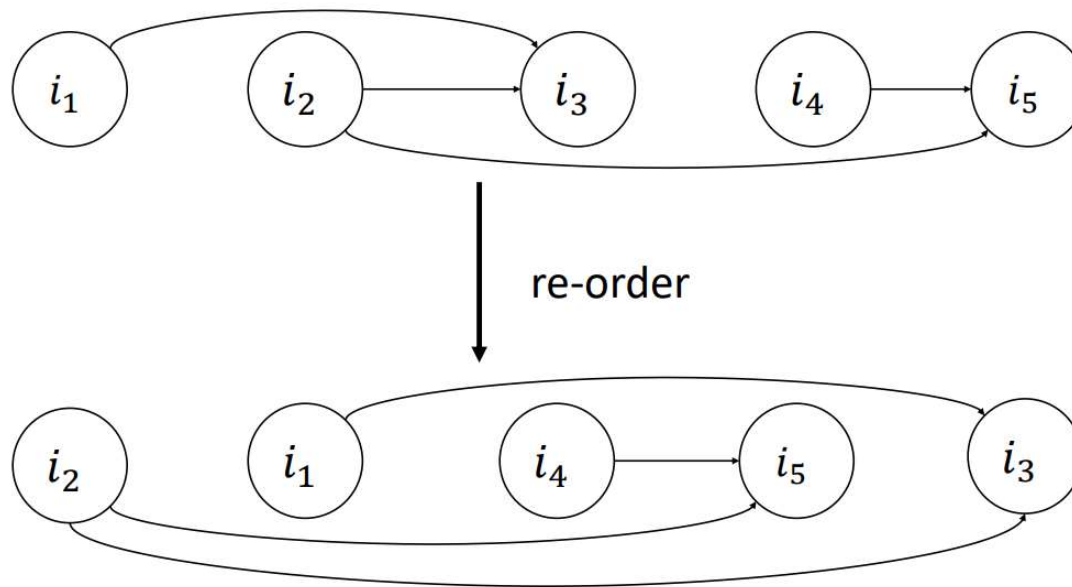
**Topological order:** given a list  $l$  and a partial order  $R$  on its elements  $E_l$ ,  $l$  is said to be an topologically sorted list by  $R$  if  $\forall i_1 i_2 \in N, R l[i_1]l[i_2] \rightarrow i_1 < i_2$  ( $l[i]$  means the  $i$ -th element of  $l$ )



a partial order  $R$  from dependence relation

a **topo-sorted** list

**Topological reorder:** Given a topo-sorted list  $l$  of elements  $A$  by  $R$ , another list  $l'$  is said to be a topo-reorder of  $l$  iff  $l'$  contains exactly the same elements as  $l$  and is also topo-sorted by  $R$ .



# Scheduling heuristics for Risc-V: an engineering trick

**Coq-OCaml interface:** a scheduling heuristics that **only affect performance, not correctness**, was not implemented in Coq but directly in an OCaml function

```
Require Import ExtrOcamlIntConv.
Parameter prioritizer : list int -> int -> list (list int) -> int -> (list int).
...
(* definition of encoding of instruction to an integer *)
...
Definition prioritizer' (l: list instruction): list positive :=
  let nodes := block2ids l in
  let edges := nblock2edges (numlistgen l) in
  let prior' := prioritizer nodes (int_of_nat (length nodes))
    edges (int_of_nat (length edges)) in
```

- Since the CompCert's Coq code will eventually be extracted to OCaml, this does not change the ***trusted computing base***

# Scheduling heuristics for Risc-V: an engineering trick

**OCaml-C interface:** the scheduling heuristics in OCaml actually uses C interface further to reduce the developing time (we have existing tools in C)

```
open Ctypes
(* The prioritizer function in OCaml *)
let prioritizer nodes n edges m: int list =
  (* First, we will need to convert them to C arrays *)
  let nodes_arr = CArray.of_list int nodes in
  let edges_arr =
    let inner = List.map (fun e -> CArray.of_list int e |> CArray.start) edges in
    let outer = CArray.of_list (ptr int) inner in outer
  in
  (* Now, we pass arguments into prioritizer *)
  let result =
    C.Functions.prioritizer (CArray.start nodes_arr) n (CArray.start edges_arr) m
  in
  CArray.from_ptr result n |> CArray.to_list
```

```
int *prioritizer(int *nodes, int n, int **edges, int m);
```

# Scheduling heuristics for Risc-V : an engineering trick

**Using Coq-OCaml-C interface is just an engineering choice, not a necessity of our implementation**

We can still do everything in Coq, but it will increase the learning burden of both *proof engineer* and *compiler-backend engineer* without improving the correctness result.